# Analysis of thermodynamic behaviour of bulk metallic glass forming melts and glass forming ability

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**Abstract** The thermodynamic behaviour of bulk metallic glass (BMG) forming melts have been studied by analyzing the temperature dependence of the Gibbs free energy difference ( $\Delta G$ ), entropy difference ( $\Delta S$ ) and enthalpy difference ( $\Delta H$ ) between the undercooled melt and the corresponding equilibrium solid phases. The study is made by calculating  $\Delta G$ ,  $\Delta S$  and  $\Delta H$  in the entire temperature range  $T_{\rm m}$  (melting temperature) to  $T_{\rm g}$  (glass transition temperature) using the expressions obtained on the basis of Taylor's series expansion. The entire analysis is made for La-based five samples of BMGs; La<sub>55</sub>Al<sub>25</sub>Ni<sub>20</sub>, La<sub>55</sub>Al<sub>25</sub>Ni<sub>15</sub>Cu<sub>5</sub>, La<sub>55</sub> Al<sub>25</sub>Ni<sub>10</sub>Cu<sub>10</sub>, La<sub>55</sub>Al<sub>25</sub>Ni<sub>5</sub>Cu<sub>15</sub>, and La<sub>55</sub>Al<sub>25</sub>Ni<sub>5</sub>Cu<sub>10</sub>Co<sub>5</sub> and a comparative study is also performed between present results and results obtained in the framework of expressions proposed by earlier workers. An attempt has also been made to study the glass forming ability for BMGs.

**Keywords** Bulk metallic glasses · Gibbs free energy · Glass forming ability · Thermodynamics of undercooled liquids

### Introduction

The advent of a new family of bulk metallic glasses (BMGs) has prompted new interest for the understanding of noncrystalline materials due to their promising technological applications. It provides a wide supercooled liquid region

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P. K. Singh e-mail: om.pavansingh@gmail.com and high thermal stability against crystallization, which offers a large experimentally accessible time and temperature window to investigate nucleation and growth in glass forming metallic melts. The studies of the thermodynamic behaviour of glass forming liquids are essential for understanding the origins of glass forming ability of materials. The Gibbs free energy change  $\Delta G$  on crystallization of multicomponent undercooled systems is an important parameter in understanding the nucleation and growth processes. In principle,  $\Delta G$  can be calculated by using the elementary thermodynamic procedures from the knowledge of experimentally measured values of the specific heat difference  $(\Delta C_P)$  between the undercooled liquid and corresponding equilibrium solid phases. However, the metastability of undercooled liquid precludes the measurement of specific heat of supercooled phase over the temperature range of interest. In absence of these data, several investigators [1–10] prompted to suggest appropriate expressions for  $\Delta G$  in terms of more easily measured parameters such as enthalpy of fusion  $\Delta H_{\rm m}$ , melting temperature  $T_{\rm m}$  etc. Most of these expressions are based on the various kinds of assumptions related to either temperature dependence of  $\Delta C_p$  or its magnitude. Recently, Lele et al. [11] obtained an expression for  $\Delta G$  using Taylor's series expansion, which has been successfully employed [12, 13] to calculate  $\Delta G$  for organic, oxide, polymeric and simple glasses. Following the earlier work of Lele et al. [11], the aim of the present work is to study  $\Delta G$  for bulk metallic glasses (BMGs) in the temperature range  $T_m$  to glass transition temperature  $(T_g)$ . The enthalpy difference  $\Delta H$  and entropy difference  $\Delta S$ between undercooled liquid and corresponding equilibrium solid phases have also been studied. The entire study is made for La-based five samples of BMGs; La<sub>55</sub>Al<sub>25</sub>Ni<sub>20</sub>, La<sub>55</sub>Al<sub>25</sub>Ni<sub>15</sub>Cu<sub>5</sub>, La<sub>55</sub>Al<sub>25</sub>Ni<sub>10</sub>Cu<sub>10</sub>, La<sub>55</sub>Al<sub>25</sub>Ni<sub>5</sub>Cu<sub>15</sub>, and La<sub>55</sub>Al<sub>25</sub>Ni<sub>5</sub>Cu<sub>10</sub>Co<sub>5</sub>. A comparative study is also made

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between present results and results obtained using the expressions proposed by earlier authors.

Glass forming ability (GFA) is an important factor for understanding the origin of glass formation and it is a very crucial parameter for designing and developing new BMGs. As a result, a number of attempts [14-20] have been made to predict GFA among the BMGs on the basis of various characteristic temperatures measured [21–23]. The most commonly used are the reduced glass transition temperature  $T_{\rm rg} = T_{\rm g}/T_{\rm l}$  and supercooled liquid region  $\Delta T_{\rm rg} = (T_{\rm x} - T_{\rm rg})^2$  $T_{\rm g}/T_{\rm l}$ , where  $T_{\rm l}$  is the liquidus temperature and  $T_{\rm x}$  is the onset crystallization temperature. Some of the workers [17] have also suggested to use  $\Delta G$  as an indicator for GFA. Dubey et al. [24, 25] have also attempted to discuss the GFA of simple glasses in terms of reduced Kauzmann [26] temperature  $(T_k/T_m)$  and  $\Delta C_p^m / \Delta S_m$ , where  $T_k$  is the Ka-uzmann temperature,  $\Delta C_p^m$  is the specific heat difference between liquid and solid phases at  $T_{\rm m}$  and  $\Delta S_{\rm m}$  is the entropy of fusion. Following the earlier work of Dubey and his coworkers [24, 25], an attempt has been made to discuss the GFA for BMGs on the basis of reduced Kauzmann temperature  $\delta = T_{\rm k}/T_{\rm m}$ ,  $\Delta C_{\rm p}^{\rm m} / \Delta S_{\rm m}$  and  $(T_{\rm g} - T_{\rm k})/T_{\rm m}$ .

# Expressions for the thermodynamic parameters $\Delta G$ , $\Delta S$ and $\Delta H$

The Gibbs free energy difference  $\Delta G$  between the undercooled liquid and corresponding equilibrium solid phases can be evaluated using thermodynamic relations

$$\Delta G = \Delta H - T \Delta S \tag{1}$$

where,

$$\Delta S = \Delta S_{\rm m} - \int_{T}^{T_{\rm m}} \frac{\Delta C_{\rm P}}{T} {\rm d}T, \qquad (2)$$

$$\Delta H = \Delta H_{\rm m} - \int_{T}^{T_{\rm m}} \Delta C_{\rm P} \mathrm{d}T \tag{3}$$

 $\Delta H_{\rm m} = T_{\rm m}\Delta S_{\rm m}$  and  $\Delta C_{\rm P} = C_{\rm P}$  (liquid)  $- C_{\rm P}$  (solid),  $C_{\rm P}$  (liquid) and  $C_{\rm P}$  (solid) are the heat capacities of the undercooled liquid and solid phases, respectively. The experimental data of  $\Delta C_{\rm P}$  enable one to use Eqs. (1–3) to evaluate  $\Delta G$  as well as other thermodynamic parameters  $\Delta S$  and  $\Delta H$ . However, as stated earlier the metastability of liquid precludes the experimental determination of  $\Delta C_{\rm P}$ . Because of these experimental difficulties, an analytical expression is needed for the evaluation of  $\Delta G$ .

Following Lele et al. [11], an expression for  $\Delta G$  can be achieved by expanding the free energies of the phases viz.,

undercooled liquid and solid around their values at  $T_{\rm m}$  in the form of Taylor's series expansion

$$G^{i} = G^{i}_{m} - \left| \frac{\partial G^{i}}{\partial T} \right|_{T_{m}} \Delta T + \frac{1}{2!} \left| \frac{\partial^{2} G^{i}}{\partial T^{2}} \right|_{T_{m}} \Delta T^{2} - \frac{1}{3!} \left| \frac{\partial^{3} G^{i}}{\partial T^{3}} \right|_{T_{m}} \Delta T^{3} + \frac{1}{4!} \left| \frac{\partial^{4} G^{i}}{\partial T^{4}} \right|_{T_{m}} \Delta T^{4} - \dots$$

$$(4)$$

where i stands to *L* and *S* to represent undercooled liquid and equilibrium solid phases, respectively, and derivatives are taken at  $T = T_m$  and  $\Delta T = T_m - T$  is the degree of undercooling. The expansion of series extends the free energy versus temperature (*G* vs. *T*) curves to each phase into their metastable regions. The proper substitution of the various thermodynamic derivatives of  $G^L$  and  $G^S$ , Eq. 4 yields

$$\Delta G = \Delta S_{\rm m} \Delta T - \frac{\Delta C_{\rm p}^{\rm m}}{(T_{\rm m} + T)} \Delta T^2 + \left| \frac{\partial \Delta C_{\rm p}}{\partial T} \right|_{T_{\rm m}} \frac{\Delta T^3}{(T_{\rm m} + T)} - \dots$$
(5)

During the analysis it is also found that the contributions due to third and other higher order terms are very small in comparison to the contributions due to the first and second terms and these can be neglected without much loss of accuracy. Consequently, a very simple expression for  $\Delta G$  can be obtained as

$$\Delta G = \Delta S_{\rm m} \Delta T - \frac{\Delta C_{\rm p}^{\rm m} \Delta T^2}{(T_{\rm m} + T)},\tag{6}$$

which requires a knowledge of  $\Delta C_p^m$ ,  $\Delta S_m$  and  $T_m$  and these parameters can be measured experimentally without much complications.

A similar Taylor's series expansion can also be carried out to get expressions for  $\Delta S$  and  $\Delta H$  and resulting expressions take the form of

$$\Delta S = \Delta S_{\rm m} - 2\Delta C_{\rm p}^{\rm m} \frac{\Delta T}{(T_{\rm m} + T)} \tag{7}$$

and

$$\Delta H = \Delta H_{\rm m} - \Delta C_{\rm p}^{\rm m} \Delta T \tag{8}$$

# $\Delta G$ , $\Delta S$ and $\Delta H$ of La-based bulk metallic glasses

The experimental values of  $\Delta G$  have been calculated with the aid of experimentally measured values of  $\Delta C_p$  reported by earlier workers [17, 27] as

$$\Delta C_{\rm p} = AT + BT^{-2} + CT^2 \tag{9}$$

with the help of Eq. 1, where A, B and C are constants. The material parameters used in the present analysis are reported

Parameters	Materials							
	La55Al25Ni20	La55Al25Ni15Cu5	La55Al25Ni10Cu10	La55Al25Ni5Cu15	La55Al25Ni5Cu10Co5			
A/J mol <sup>-1</sup> K <sup>-2</sup>	$2.194 \times 10^{-2}$	$2.57 \times 10^{-2}$	$3.445 \times 10^{-2}$	$2.823 \times 10^{-2}$	$2.522 \times 10^{-2}$			
$B/J mol^{-1} K^{-3}$	$1.235 \times 10^{6}$	$1.078 \times 10^{6}$	$1.463 \times 10^{6}$	$1.666 \times 10^{6}$	$1.687 \times 10^{6}$			
$C/J \text{ mol}^{-1} \text{ K}^{-2}$	$-1.009 \times 10^{-5}$	$-9.208 \times 10^{-6}$	$-3.663 \times 10^{-5}$	$-2.202 \times 10^{-5}$	$-1.813 \times 10^{-5}$			
$\Delta H_{\rm m}/{ m J}~{ m mol}^{-1}$	$7.48 \times 10^{3}$	$7.51 \times 10^{3}$	$6.838 \times 10^{3}$	$7.21 \times 10^{3}$	$6.095 \times 10^{3}$			
$\Delta S_{\rm m}/{ m J}~{ m mol}^{-1}~{ m K}^{-1}$	10.512	11.384	10.328	10.868	9.222			
$\Delta C_p^m/\mathrm{J} \mathrm{mol}^{-1} \mathrm{K}^{-1}$	12.942	15.464	10.089	12.822	12.611			
T <sub>m</sub> /K	711.6	659.7	662.1	663.4	660.9			
T <sub>g</sub> /K	490.5	471.9	467.4	459.1	465.7			

**Table 1** The material parameters used for the evaluation of  $\Delta G$ ,  $\Delta S$  and  $\Delta H$  of five samples of La-based BMGs



**Fig. 1** The variations of  $\Delta G$ ,  $\Delta H$  and  $\Delta S$  with  $\Delta T$  for La<sub>55</sub>Al<sub>25</sub>Ni<sub>20</sub> in the temperature range  $T_{\rm m}$  to  $T_{\rm g}$ . Solid lines are calculated values while *triangles, closed circles* and *open circles* stand for experimental values of  $\Delta G$ ,  $\Delta H$  and  $\Delta S$ , respectively

in Table 1. To see the applicability of expression for  $\Delta G$  stated in Eq. 6,  $\Delta G$  has been calculated for five samples of La-based BMGs; La<sub>55</sub>Al<sub>25</sub>Ni<sub>20</sub>, La<sub>55</sub>Al<sub>25</sub>Ni<sub>15</sub>Cu<sub>5</sub>, La<sub>55</sub>Al<sub>25</sub>Ni<sub>10</sub>Cu<sub>10</sub>, La<sub>55</sub>Al<sub>25</sub>Ni<sub>5</sub>Cu<sub>15</sub> and La<sub>55</sub>Al<sub>25</sub>Ni<sub>5</sub>Cu<sub>10</sub>Co<sub>5</sub> in the entire temperature range  $T_{\rm m}$  to  $T_{\rm g}$  and results obtained are illustrated in Figs. 1, 2, 3, 4, 5. The temperature dependence of thermodynamic parameters  $\Delta S$  and  $\Delta H$  has also been studied by estimating  $\Delta S$  and  $\Delta H$  for all samples using Eqs. 7 and 8, respectively, in the temperature range  $T_{\rm m}$  to  $T_{\rm g}$  and results obtained are also illustrated in Figs. 1, 2, 3, 4, 5 of the respective samples. The experimental values of  $\Delta S$  and  $\Delta H$  are evaluated using Eqs. 2 and 3, respectively.

From Figs. 1, 2, 3, 4, 5, it can be seen that the agreement between calculated and experimental values of  $\Delta G$  is very good for all five samples in the entire temperature range  $T_{\rm m}$ to  $T_{\rm g}$  and it can be said that expression reported in Eq. 6 is



Fig. 2 The variations of  $\Delta G$ ,  $\Delta H$  and  $\Delta S$  with  $\Delta T$  for La<sub>55</sub>Al<sub>25</sub>. Ni<sub>15</sub>Cu<sub>5</sub> in the temperature range  $T_{\rm m}$  to  $T_{\rm g}$ . The solid lines represent calculated values and *triangles*, *closed circles* and *open circles* are experimental values of  $\Delta G$ ,  $\Delta H$  and  $\Delta S$ , respectively

capable of explaining temperature dependence of  $\Delta G$  very successfully for BMG samples. From these Figures, it can also be seen that the agreement between calculated and experimental values of  $\Delta S$  and  $\Delta H$  is very good for low degree of undercooling while there are some discrepancies for three samples La<sub>55</sub>Al<sub>25</sub>Ni<sub>10</sub>Cu<sub>10</sub>, La<sub>55</sub>Al<sub>25</sub>Ni<sub>5</sub>Cu<sub>15</sub> and La<sub>55</sub>Al<sub>25</sub>Ni<sub>5</sub>Cu<sub>10</sub>Co<sub>5</sub> for large degree of undercooling. Thus one can say that the expressions for  $\Delta S$  and  $\Delta H$  reported in Eqs. 7 and 8, respectively, are also quite capable to explain the temperature dependence of  $\Delta S$  and  $\Delta H$ .

A comparative study has also been made between the present result and the results obtained in the frame of expressions proposed by earlier workers by calculating  $\Delta G$  for one sample La<sub>55</sub>Al<sub>25</sub>Ni<sub>5</sub>Cu<sub>10</sub>Co<sub>5</sub> and results obtained are reported in Table 2. From Table 2, it can be seen that



Fig. 3 The variations of  $\Delta G$ ,  $\Delta H$  and  $\Delta S$  with  $\Delta T$  for La<sub>55</sub>Al<sub>25</sub>. Ni<sub>10</sub>Cu<sub>10</sub> in the temperature range  $T_{\rm m}$  to  $T_{\rm g}$ . Solid lines represent calculated values while *triangles*, *closed circles* and *open circles* show experimental values of  $\Delta G$ ,  $\Delta H$  and  $\Delta S$ , respectively



**Fig. 4** The variations of  $\Delta G$ ,  $\Delta H$  and  $\Delta S$  with  $\Delta T$  for La<sub>55</sub>Al<sub>25</sub>. Ni<sub>5</sub>Cu<sub>15</sub> in the temperature range  $T_{\rm m}$  to  $T_{\rm g}$ . Solid lines correspond to calculated values while *triangles*, *closed circles* and *open circles* represent experimental values of  $\Delta G$ ,  $\Delta H$  and  $\Delta S$ , respectively

the values of  $\Delta G$  obtained on the basis of expression stated in Eq. 6 based on Taylor's series expansion are closest to the experimental values even for large degree of undercoolings. At  $T = T_g$ , the difference between the experimental and calculated values of  $\Delta G$  is only about 0.44% for the undercooling  $\Delta T = 195.2$  K while it is 31.58%, 7.3%, 8.77%, 28.0%, 10.09% and 1.75% on the basis of



**Fig. 5** The variations of  $\Delta G$ ,  $\Delta H$  and  $\Delta S$  with  $\Delta T$  for La<sub>55</sub>Al<sub>25</sub>Ni<sub>5-</sub>Cu<sub>10</sub>Co<sub>5</sub> in the temperature range  $T_{\rm m}$  to  $T_{\rm g}$ . Solid lines show the calculated results while *triangles, closed circles* and *open circles* are experimental values of  $\Delta G$ ,  $\Delta H$  and  $\Delta S$ , respectively

expressions reported by Turnbull [1], Hoffmann [2], Thompson and Spaepen [4], Singh and Holtz [5], Lad et al. [6] and Dhurandhar et al. [9], respectively.

Thus, the expression for  $\Delta G$  stated in Eq. 6 can be used to study the nucleation and growth processes of bulk metallic glass forming melts. At the same time, it is needed to state that expression for  $\Delta G$  used is quite simple and needs  $\Delta C_p^m$ ,  $\Delta S_m$  and  $T_m$  as input data which can be measured easily.

# **Glass forming ability of BMGs**

Kauzmann [26] first of all, who studied the importance of rate of entropy loss with temperature in liquids and pointed out that a liquid loses its entropy at a faster rate than the solid resulting in the equal entropy of both phases solid and liquid at some temperature  $T_k$  well above the absolute temperature. The iso-entropic temperature  $T_k$  is referred as the ideal glass transition temperature. It is also called the Kauzmann temperature and it plays an important role in the estimation of residual entropy  $\Delta S_R$  and GFA of materials. Realizing that  $\Delta S = 0$  at  $T = T_k$ , and using Eq. 2, the Kauzmann temperature  $T_k$  has been estimated for all five samples and results obtained are reported in Table 3.

Kauzmann predicted that the rate of change of reduced entropy  $\Delta S/\Delta S_{\rm m}$  with respect to reduced temperature  $T/T_{\rm m}$ measures the likelihood that  $T_{\rm k}$  exists above the absolute zero. He further argued that the derivative attains a value  $\Delta C_{\rm p}^{\rm m} / \Delta S_{\rm m}$  at  $T_{\rm m}$ . Consequently, high value of  $\delta (= T_{\rm k}/T_{\rm m})$ can be anticipated for materials with larger values of

**Table 2** The Gibbs Free energy difference  $\Delta G$  between undercooled liquid and corresponding equilibrium solid phases of La<sub>55</sub>Al<sub>25</sub>Ni<sub>5</sub>Cu<sub>10</sub>Co<sub>5</sub> calculated using various expressions of  $\Delta G$  given by earlier workers. Experimental value of  $\Delta G$  is evaluated using experimental data of  $\Delta C_p$ 

<i>T</i> /K	Δ1/Κ	$\Delta G$ (J/mol)								
		Turnbull [1]	Hoffman [2]	Thompson and Spaepen [4]	Singh and Holtz [5]	Lad et al. [6]	Dhurandhar et al. [9]	Present	Experimental	
465.7 (T <sub>g</sub> )	195.2	1800	1268	1488	5209	1230	1344	1374	1368	
475	185.9	1714	1232	1434	5018	1199	1306	1331	1326	
500	160.9	1484	1123	1278	4474	1101	1188	1203	1200	
525	135.9	1253	996	1110	3884	983	1048	1057	1056	
550	110.9	1023	851	929	3252	844	890	895	894	
575	85.9	792	689	737	2580	686	715	717	717	
600	60.9	562	510	535	1871	509	524	524	524	
625	35.9	331	313	322	1126	313	318	318	318	
650	10.9	101	99	100	349	99	99	99	99	

Table 3 The parameters used in analysis of GFA of BMGs

S·N.	Materials	Materials parameters						
		T <sub>k</sub> /K	$T_{\rm g} - T_{\rm k}/{\rm K}$	$\frac{\Delta C_p^m}{\Delta S_m}$	$\delta=rac{T_{ m k}}{T_{ m m}}$	$\left(\frac{T_{g}-T_{k}}{T_{m}}\right)$	$R_{\rm c}/{\rm K~s}^{-1}$	
1	La55Al25Ni20	335.869	154.631	1.231	0.472	0.217	67.5	
2	La55Al25Ni15Cu5	317.497	154.403	1.358	0.481	0.234	34.5	
3	La55Al25Ni10Cu10	332.019	135.381	0.977	0.501	0.218	22.5	
4	La55Al25Ni5Cu15	340.031	119.069	1.179	0.513	0.179	35.9	
5	La55Al25Ni5Cu10Co5	362.93	102.77	1.367	0.549	0.516	18.8	



Fig. 6 The variation of  $\delta$  with  $\Delta C_p^m / \Delta S_m$  for BMGs. The numbers in the plot refers to the corresponding material as indicated serial numbers in Table 3

 $\Delta C_p^m / \Delta S_m$  and such a correlation can be seen in Fig. 6 which shows the variation of  $\delta$  with  $\Delta C_p^m / \Delta S_m$ . The actual relationship between  $\delta$  and  $\Delta C_p^m / \Delta S_m$  requires the knowledge of exact temperature dependence of  $\Delta S$  which can be arrived at only from the knowledge of  $\Delta C_{\rm p}$ . However, an empirical relation can be obtained as

$$\delta = a_1 + b_1 \left(\frac{\Delta C_p^{\rm m}}{\Delta S_{\rm m}}\right)^{m_1} \tag{10}$$

with  $a_1 = 0.4977$ ,  $b_1 = 0.0040$  and  $m_1 = 8.1822$  and correlation factor 0.9998.

The critical cooling rate  $R_c$  can be used for quantifying the glass forming ability of bulk metallic glasses. The smaller  $R_c$ , the higher the GFA of a material and such melts are known as better glass former. Dubey and Ramachandrarao [24] studied the GFA of simple glasses and reported the crucial role of  $\delta$  and  $\Delta C_p^m / \Delta S_m$  in the estimation of  $R_c$ . Thus, it is interesting to study the role of these parameters in the analysis of GFA of BMGs.

The variation of  $R_c$  [28] with  $\delta$  has been illustrated in Fig. 7 which shows decreasing nature of  $R_c$  with increasing  $\delta$ . It may be argued that materials with a large value of  $\delta$ are likely to form glasses more easily by requiring a low  $R_c$ . It is needed to state that Turnbull [29] as well as Davies [30] have attempted to correlate  $T_g/T_m$  with  $R_c$ . It is very difficult to have an analytical relation between  $R_c$  and  $\delta$ . However, an attempt has been made to have a simple relation based on experimental data as



Fig. 7 The variation of critical cooling rate  $R_c$  with  $\delta$  for BMGs. The numbers in the plot indicates the corresponding material as the serial numbers in Table 3



Fig. 8 The variation of critical cooling rate  $R_c$  with  $\Delta C_p^m / \Delta S_m$  for BMGs. The numbers in the plot is the corresponding material as the serial numbers Table 3

$$\log R_c = a_2 + b_2 \delta^{m_2} \tag{11}$$

with  $a_2 = 19.88$ ,  $b_2 = 5.34$  and  $m_2 = -61.28$  and correlation factor 0.9978.

As seen above,  $\delta$  is in self a function of  $\Delta C_p^m / \Delta S_m$ . Hence,  $R_c$  should correlate with  $\Delta C_p^m / \Delta S_m$ . The variation of  $R_c$  with  $\Delta C_p^m / \Delta S_m$  is illustrated in Fig. 8 which shows the decreasing nature of  $R_c$  with increasing  $\Delta C_p^m / \Delta S_m$ 



Fig. 9 The variation of critical cooling rate  $R_c$  with  $(T_g - T_k)/T_m$  for BMGs. The numbers in the plot refers to the corresponding material of the serial numbers as listed in Table 3

except for the sample La<sub>55</sub>Al<sub>25</sub>Ni<sub>10</sub>Cu<sub>10</sub>. It can be understood by looking the correlation between  $\delta$  and  $\Delta C_p^m / \Delta S_m$ . Thus, once again it may be seen that the material having large  $\Delta C_p^m / \Delta S_m$  requires low  $R_c$  and such material can be converted into glassy state more easily. A simple relation can be obtained on the basis of data shown in Fig. 8 as

$$\log R_{\rm c} = a_3 + b_3 \left(\frac{\Delta C_{\rm p}^{\rm m}}{\Delta S_{\rm m}}\right)^{m_3} \tag{12}$$

with  $a_3 = -14522.93586$ ,  $b_3 = 14676.17534$  and  $m_3 = -0.02805$  and correlation factor 0.9314.

Both the Kauzmann temperature  $T_k$  and the glass transition temperature  $T_{g}$  play a crucial role in viscosity of glass forming melts as well as nucleation and growth processes. As a result, both  $T_{\rm g}$  and  $T_{\rm k}$  play a significant role in assigning the GFA of BMGs and it is interesting to study the combined effect of these parameters in the study of GFA of glass forming melts. The variation of R<sub>c</sub> with  $(T_{\rm g} - T_{\rm k})/T_{\rm m}$  is illustrated in Fig. 9 for all five samples which shows decreasing nature of  $R_c$  with decreasing  $(T_{\rm g} - T_{\rm k})/T_{\rm m}$ . It means, the material having  $T_{\rm k}$  closer to  $T_{\rm g}$ requires low degree of undercooling to convert into glassy state and such material is known as good glass former. It can also be understood in term of residual entropy or frozen-in-entropy  $\Delta S_{\rm R}$ . According to Gibbs and Dimarzio [31], the residual entropy or frozen-in-entropy is the amount of entropy of the glass forming melt that has been blocked in at  $T_{g}$  during the glass formation and remains in the glassy state even at T = 0 K. The materials having low value of  $(T_{\rm g} - T_{\rm k})$  corresponds to low value of residual entropy. Thus the material showing low  $(T_g - T_k)/T_m$  can

form glasses more readily than those having large  $(T_{\rm g} - T_{\rm k})/T_{\rm m}$ .

An attempt has also been made to establish a relation between  $R_c$  and  $(T_g - T_k)/T_m$  on the basis of data reported in Table 3, and it can be expressed as

$$\log R_{\rm c} = a_4 + b_4 \left(\frac{T_{\rm g} - T_{\rm k}}{T_{\rm m}}\right)^{m_4} \tag{13}$$

where,  $a_4 = -1.03596 \times 10^5$ ,  $b_4 = 1.03890 \times 10^5$  and  $m_4 = 1.43 \times 10^{-3}$  and correlation factor R = 0.9945.

### Conclusion

The thermodynamic behaviour of BMGs have been studied by estimating  $\Delta G$ ,  $\Delta S$  and  $\Delta H$  for five samples of La-based bulk metallic glasses using expressions obtained on the basis of Taylor's series expansion and results obtained are very close to the experimental values. The value of  $\Delta G$ obtained in the present work is better than  $\Delta G$  obtained in the frame of expression reported by earlier workers.

The ideal glass transition temperature  $T_{\rm k}$  has been estimated and GFA of BMGs has been studied on the basis of  $\delta$ ,  $\Delta C_{\rm p}^{\rm m} / \Delta S_{\rm m}$  and  $(T_{\rm g} - T_{\rm k})/T_{\rm m}$ . It has been found that material having large  $\Delta C_{\rm p}^{\rm m} / \Delta S_{\rm m}$  shows a large  $\delta$  and such material requires a low  $R_{\rm c}$  for glass formation. Thus material having large  $\Delta C_{\rm p}^{\rm m} / \Delta S_{\rm m}$  can be said as good glass former. It is further found that material showing  $T_{\rm k}$  closer to  $T_{\rm g}$  requires low  $R_{\rm c}$  for glass formation and such material can be said a better glass former.

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